ISI – Bangalore Center – B Math - Physics I – End Semestral Exam
Date: 6 May 2015. Duration of Exam: 3 hours
Total marks: 50
ANSWER ALL QUESTIONS

Q 1 [Total Marks: 10]
FOR Q1, only the final answer needs to be given; Calculation to get the final answer can be done in the rough pages but need not be shown.

Q 1a. A rigid thin circular ring lies on the surface of a smooth fixed (stationary) sphere whose radius is greater than that of the ring. The forces of constraints are such that the every point of the ring always stays in contact with the surface of the sphere. Otherwise the ring is free to move. How many generalized coordinates are required to specify the motion of the ring? [2]

Q 1b. A sphere of mass *m* subjected to the earth's gravitational field is rotating with angular speed ω around a fixed vertical diameter. A small particle of mass *m/5* is originally at rest at the north pole (the north pole is the top of the axis of rotation). Due to a short infinitesimal disturbance it begins to slide down along a great circle. Let the angular speed be ω' when the particle has reached half way towards the equator. What is ω' / ω? Take the center of the sphere as the origin. Assume no friction and that there is no component of the total torque along the axis of rotation. [3]

Q1c. Point out two distinct reasons because of which the following matrix $\begin{pmatrix} 4 & 3 & 1 \\ 2 & -9 & 6 \\ 1 & 5 & 7 \end{pmatrix}$ cannot represent a Moment of Inertia tensor. [2]

Q1c. A one dimensional spring has a natural frequency of oscillation ω. Its motion is damped but subject to a external force $F(t) = A\,(\sin\,\omega t)(\cos\,\omega t)$. What will be the frequency of oscillation of the spring after a "long" time? [2]

Q1d. A particle of mass m moving with speed v collides inelastically with another of mass m at rest in the lab frame of reference. Let the kinetic energy lost in the collision be Q joules as measured in the lab frame. Seen in the CM frame (which is moving with speed v/2 with respect to the lab frame) the energy loss in the inelastic collision is Q'. Which of the following statements is true? [1]

$Q' = Q$, or $Q' = Q - ½\, m\, v^2$, or $Q' = Q + ½\, m\, v^2$, or none of these

Q2. [Total Marks: 10]

Consider a dumbbell made of two uniform spheres of radius b and mass M each, connected by a thin, rigid and massless rod of length $2l$. The body is rotating through some axis through its center of mass. Assume that at some instant of time t_0, the rod and coincides with the Z axis with the middle of the rod as the origin and at the same instance the angular momentum vector $\vec{\omega}$ is in the y-z plane, i.e. $\vec{\omega} = \omega_y \vec{j} + \omega_z \vec{k}$.

Recall that in general $L_i = \sum_j I_{ij}\omega_j$ where $\vec{L} = (L_x, L_y, L_z)$ and similarly $\vec{\omega} = (\omega_x, \omega_y, \omega_z)$.

Answer the following questions regarding the dumbbell at the instance of time t_0.

a. Show that $I_{ij} = 0$ for $i \neq j$. [3]

b. Express I_{zz}, in terms of M and b. [2]

c. Let $I_{yy} = I_{zz} + X$. Determine X in terms of M, l and b. [Hint: Use parallel axis theorem] [2]

d. Find the three components of L and show that L is not in the same direction as $\vec{\omega} = \omega_y \vec{j} + \omega_z \vec{k}$. [3]

Q3. [Total Marks: 10]

3a.) Show that for the angular momentum of a system to be conserved along a fixed axis, it is necessary and sufficient to have the total external torque to be orthogonal to the fixed axis.

3b.) Either prove the following statement or give a counter example: The motion of a particle subject to a central force is always confined to a two dimensional plane.

Q4. [Total Marks: 10]

A uniform rope of mass M and length a is held at rest with its two ends close together and the rope hanging symmetrically below. (In this position the rope has two long vertical segments connected by a negligibly small curved segment in the bottom.) One of the ends is released while the other end is supported at the same place.

Assume uniform tension along the rope and that the total energy is conserved in the process as the released end of the rope is falling.

4a.) Show that the speed of the free end when it has descended by a distance x is given by $v^2 = \frac{xg(2a-x)}{a-x}$

4b.) Find the reaction R exerted by the support at the end held fixed y when the free end has descended by a distance x.

4c.) If the support collapses when R exceeds (3/2)Mg, then how far will the free end fall before the support collapses?

Q5. [Total Marks: 10]

5a.) Show that the path y= y(x) for which the integral $\int_{x1}^{x_2} x\sqrt{1-y'^2}\,dx$ is stationary is a sinh function.

5b.) Two particles of each with mass m are constrained to move in a two dimensional plane under mutual interaction that can be derived from a potential $U(|\vec{r}_1 - \vec{r}_2|)$ where $\vec{r}_1 = (x_1, y_1), \vec{r}_2 = (x_2, y_2)$.

Write the Lagrangian L as a function of $(x_1, y_1, x_2, y_2, \dot{x}_1, \dot{y}_1, \dot{x}_2, \dot{y}_2)$.

Write the same Lagrangian as a function of $(X, Y, r, \theta, \dot{X}, \dot{Y}, \dot{r}, \dot{\theta})$, where

$$X = \frac{x_1 + x_2}{2}, Y = \frac{y_1 + y_2}{2}, r\cos\theta = \frac{x_1 - x_2}{2}, r\sin\theta = \frac{y_1 - y_2}{2}.$$

Find the cyclic coordinates in $L(X, Y, r, \theta, \dot{X}, \dot{Y}, \dot{r}, \dot{\theta})$ (i.e. the coordinates L does not depend on) find the corresponding conservations laws.

--------------------------Useful Information----------------

The moment of inertia of a solid sphere is $\frac{2}{5}Mr^2$ using obvious notations and conventions.